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Decision Analysis in Forensic Science

ABSTRACT: Forensic scientists are routinely faced with the problems of making decisions under circumstances of uncertainty (i.e., to perform or not perform a test). A decision making model in forensic science is proposed, illustrated with an example from the field of forensic genetics. The approach incorporates available evidence and associated uncertainties with the assessment of utilities (or desirability of the consequences). The paper examines a general example for which identification will be made of the decision maker, the possible actions, the uncertain states of nature, the possible source of evidence and the kind of utility assessments required. It is argued that a formal approach can help to clarify the decision process and give a coherent means of combining elements to reach a decision.

KEYWORDS: forensic science, decision analysis, Bayes theorem, scientific evidence, evaluation, interpretation, kinship determination, utility

An evaluation process starts when the scientist first meets the case. It is at this stage that the scientist thinks about the questions that are to be addressed and the outcomes that may be expected. The scientist should attempt to frame propositions of interest and think about the weight of evidence that is expected (1). There is a wide tendency to consider evaluation of evidence as a final step of a casework examination, notably at the time of preparing the formal report. This is so even if an earlier interest in the process would enable the scientist to make better decisions about the allocation of resources. A first approach to decision-making in an operational forensic science problem has been proposed by Cook et al. (2). It is based on a model embodying the principle of likelihood ratio as a measure of the weight of evidence. In that spirit, (2) proposed a model for enhancing the cost-effectiveness of a casework activity from initial contact with the customer. The aim is to enable the customer to make better decisions.

In routine work, an estimate of the expected likelihood ratio is often requested by forensic genetics laboratories, before the performance of any blood tests. Such an estimate will help the scientist to support a better decision for the customer.

Imagine a situation in paternity testing where the alleged father is unavailable but a cousin of the alleged father could potentially be considered and tested. In such a case the two propositions of interest may be of the form of:

H_p: The tested person is a cousin of the true father,
H_d: The tested person is unrelated to the child.

Two questions are of interest: (1) can we obtain a value supporting the hypothesis H_p or H_d in this scenario?, and (2) how can the laboratory or the customer take a rational decision on the necessity to perform blood tests after an estimate of possible values of likelihood ratio?

The first question refers to the *pre-assessment* process, the second to *decision making*. Cook et al. (2) proposed answers to the first question.

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In the early 1950's, a debate was initiated as to how people should make decisions involving money that were in some sense rational, and as to how people in fact made monetary decisions, and whether these could be regarded as rational (3). This debate—through the use of the notion of utility function—was extended in the context of financial decision-making. This paper proposes these ideas in a forensic context involving scientific evidence. In forensic science, decision theory can be used to develop general approaches to determining optimal choices given a certain body of evidence and values. The aims of this paper are the development of earlier works on pre-assessment and the provision of an answer to the second question using decision theory. The perspective will be that of an individual decision-maker, who is either the customer or acts on behalf of the customer (i.e., the forensic scientist) and is interested in the determination of an optimal course of action using formal modeling.

Graphical models are also introduced briefly to deal with the decision problem of interest. They are intended to support human reasoning and decision making through the formalization of expert knowledge (4).

Pre-assessment

A scientist requires an adequate appreciation of the circumstances of the case so that a framework may be set up for consideration of the kind of examination that may be carried out and what may be expected from them, in order for a logical procedure to be performed (2).

The choice of level (e.g., activity level rather than source level; see (5) for a discussion of levels) for the propositions, H_p and H_d, for the evaluation of scientific evidence is carried out within a framework of circumstances, and these circumstances have to be known before any examination of the evidence is made in order that relevant propositions may be proposed. This process provides a basis for consistency of approach by all scientists who are thereby encouraged to consider carefully factors such as circumstantial information and data that are to be used for the evaluation of evidence and to declare them in the final report.

The scientist should proceed by considering the estimation of the probability of whatever evidence will be found given each proposition. Consider, for example, a case where a window is smashed (this example is presented in (2), and assume that the prosecution and defence propose the following propositions at the activity level: "The suspect is the man who smashed the window," and "The suspect did not smash the window." The examination of the suspect's pullover will reveal a number Q of glass fragments, where Q can be, for example and for sake of simplicity, one of the following states: *none*, *few* and *many*. So:

1. The first question asked for assessment of the numerator of the likelihood ratio is "what is the probability of finding a number Q of 'matching' glass fragments if the suspect is the man who smashed the window?"
2. The second question asked for assessment of the denominator of the likelihood ratio is "what is the probability of finding a number Q of 'matching' glass fragments if the suspect is not the man who smashed the window?"

The scientist is asked initially to assess six different probabilities (three states for the variable Q and two propositions) using data coming from surveys, relevant publications on the matter or subjective assessments (6).

Note that these probabilities may not be easy to derive because of a possible lack of information available to the scientist. For example, it will be very difficult to assess transfer probabilities if the scientist has no answer to questions that concern matters such as the *modus operandi*. (How was the window smashed? If it was smashed by a person, then was that person standing close to it?) Information about the way a window is smashed is important because it provides information on the amount of glass potentially projected. Information of the distance between the window and the person who smashed it offers relevant information on the amount of glass fragments the scientist will expect to recover. It is also difficult to assess the probability of persistence of any transferred glass fragment. All of these aspects are outside the scope of this paper.

For the sake of illustration, Cook et al. (2) proposed probability distributions for finding a number, Q, of matching glass under the two competing propositions, H_p and H_d. Therefore, they were able to calculate likelihood ratios for the three states of Q as a measure of the value of the evidence Q. Table 1 summarizes their estimates.

These may lead to conclusions that:

> "[...] on the basis of this assessment, if the suspect is indeed the person who smashed the window, there is a 65% chance that the result of the examination will provide moderate support for that proposition, and a 30% chance that it will provide weak support. If, on the other hand, the suspect did not smash the window then there is a 95% chance of moderate evidence to support [this alternative proposition] although there is a 5% chance of evidence which will tend falsely to incriminate him." [(2) at 155]

From these results, it is suggested that the scientist is in a position to help the customer make a decision. This stage is a fundamental one in the process of making decisions, but it does not offer clear criteria for the decision as to whether or not to perform a test (i.e., analytical test in the laboratory).

TABLE 1—*Probabilities of Q and likelihood ratio.*

Q	$\Pr(Q\|H_p)$	$\Pr(Q\|H_d)$	V
None	0.05	0.95	0.05
Few	0.30	0.04	7.5
Many	0.65	0.01	65

Scientists should make decisions and they do so generally under uncertainty. A logical framework should be employed to reach this task.

Decision-making

The process of making a decision consists in the choice, given personal objectives, from two or more (including infinite) possible outcomes of the one that is considered as the most suitable when the consequences of the choice is uncertain.

One of the well-known rules for decision making under uncertainty is the *Arnauld* rule: we are uncertain about what will happen, or what is true, but we are also uncertain about what to do. Decisions need more than probability, they are based on the values of the possible outcomes of our actions. To be a rational decision maker, it is required to choose the decision offering the highest probability of obtaining the appropriate consequence. This rule is known as the *rule of the maximization of the expected utility* (7). De Finetti argues that this approach is the only one that guarantees that the choice (between possible actions) is coherent: either we choose using this rule or our choices are clearly irrational (8). A particular aspect of this rule has been presented by Wald (9) and it relates to the concept called the *admissibility of a decision*. A decision is admissible if there exist no better decision, that is if it is not dominated by any other decision. (A decision d dominates d' if the consequences of making d are never worst than those of making d', and are better in at least one case). Note that a decision is admissible if and only if it is coherent with the expected utility scheme.

Thus, decision theory (a theory for making decisions) provides a unified framework for integrating all aspects of a decision problem. More formally *decision theory* can be defined using Lindley's words:

> "Decision theory is a mathematical theory of how to make decisions in face of uncertainty about the true value of parameters. [...] The first element in decision theory is a set of parameters θ which describes in some way the material of interest and about which the decisions have to be made. The second element is a set of decisions d which contains all the possible decisions that might be taken. Notice that the set of decisions is supposed to contain all the decisions that could be taken and not merely some of them: or to put it differently, we have, in theory, to choose *amongst* a number of decisions. Thus it would not be a properly defined decision problem in which the only decision was whether to go to the cinema, because if the decision were not made (that is, one did not go to the cinema) one would have to decide whether to stay at home and read, or go to the public-house, or indulge in other activities. All the possible decisions, or actions, must be included in the set." [(10) at 62–63]

A first task in any decision problem is to draw up an exhaustive list of actions that are available: $d_1, d_2, \ldots, d_m \in \Delta$. The space of decisions Δ is provided with a partial pre-ordering, denoted by $\succ$; this means that it is all the time possible to detect which decision is suitable or whether they are equivalent (11). It is convenient to make the requirement of exclusivity: only one of the decisions can be selected.

Secondly, a list of n exclusive and exhaustive uncertain events (also called *states of nature*) is needed: $\theta_1, \theta_2, \ldots, \theta_n \in \Omega$, where Ω denotes the entirety of nature.

Decision theory can apply to conditions of certainty, risk or uncertainty. Decision under certainty means that each alternative leads to one and only one consequence, and a choice among alternatives is equivalent to a choice among consequences. Decision problems can be complicated because of uncertainty about what the future holds. Many important decisions have to be made without knowing exactly what it will happen in the future, in the sense that each alternative will have one of several consequences. It is possible to measure uncertainty on the events using a suitable probability distribution P over Ω. Therefore, each alternative is associated with a probability distribution and a choice among probability distributions. In decision under risk, the probability of occurrence for each consequence of any decision is known. When the probability distributions are unknown, we are faced to decision under uncertainty. Examples through this paper approach situations under risk.

Utility as a Probability

The two elements, the decision d and the event θ, are related to one another. The main problem of the two lists is to choose a member of the first list (decision) without knowing which member of the second list (state of nature) happens.

The combination of decision d_i with state of nature θ_j will result in a foreseeable consequence. This consequence will be written as $C_{ij} = C_{d_i}(\theta_j)$. Varying d_i, $i = 1, \ldots, m$, and θ_j, $j = 1, \ldots, n$, a space of consequences is obtained. Consequences are defined in such a way that it is possible for them to be ranked with the first in the ranking called "best" and the last "worst." Notice that this particular ranking is not inevitable. With this particular ranking, it is not immediately obvious which decision should be taken. It follows that the next task is to provide something more than just a ranking. In order to do this a standard is introduced and a coherent comparison (i.e., a comparison which does not produce inconsistencies) with this standard provides a numerical assessment.

Assume C is the happiest consequence and c is the worst of them. C and c being a reference pair of highly desirable and highly undesirable consequences, respectively. It follows that any consequence C_{ij} may be compared unfavourably with C and favorably with c. Associated with any consequence C_{ij} is a unique number $u \in (0, 1)$ such that C_{ij} is just desirable as a probability u of C and $1 - u$ of c. The number associated with C_{ij} will denoted $u(C_{ij})$ or $u(d_i, \theta_j)$ and will be called the *utility* of C_{ij}.

Utility is a measure of the desirability of consequences of a course of action that applies to decision-making under risk (uncertainty with known probabilities). The measure of the utility is assumed to reflect the preferences of the decision maker. The numerical order of utilities for consequences has to preserve the decision maker's preference order among the consequences. So, for example, for decision (action) d_i and three mutually exclusive states of nature, θ_1, θ_2 and θ_3, if the decision maker prefers consequence C_{13} to C_{12} and C_{12} to C_{11}, the utilities assigned must be such that $u(C_{13}) > u(C_{12}) > u(C_{11})$. In general it can be said that decision d_i dominates weakly decision d_k if and only if $u(d_i, \theta) \geq u(d_k, \theta)$ for every $\theta \in \Omega$:

$$d_i \succ d_k \Leftrightarrow u(d_i, \theta) \geq u(d_k, \theta), \quad \forall \theta \in \Omega.$$

The domination is said to be strong if $\geq$ is replaced with $>$.

This utility is a probability: u is by definition the probability to obtain the best consequence. So numbers are associated with decisions in such a way that the best decision is that with the highest number. If decision d_i is taken and if state θ_j occurs, the probability of obtaining the consequence C is $u(C_{ij})$:

$$\Pr(C|d_i, \theta_j) = u(C_{ij}).$$

The *expected utility* of decision d_i is:

$$E(U|d_i) = \sum_{j=1}^{n} \Pr(C|d_i, \theta_j) \Pr(\theta_j) = \sum_{j=1}^{n} u(C_{ij}) \Pr(\theta_j).$$

The expected utility $E(U|d_i)$ gives a numerical value to the probability of obtaining the best consequence C if decision d_i is taken. A decision problem is solved by maximizing expected utility (note that it is possible to formulate decision problem in an alternative way in terms of loss or regret associated with each pair (θ, d) by defining a loss function. The loss function $L(\theta, d)$ is the difference between the utility of the outcome of action d for state θ and the utility of the outcome of the best action for that state. Therefore, the action minimizing the expected loss is the same as the action maximizing the expected utility). The numerical order of expected utilities of actions preserves the decision maker's preference order among these actions.

Utility is not just a number describing the attractiveness of a consequence but is a number measured (from the main point of view expressed in this text) on a probability scale and obeys the laws of probability. It is a measure of the value of the decision d_i: the greater the expected utility, the greater the desirability of the decision because it offers a greater probability of obtaining the better consequence.

Note that the same result would have been obtained had other standards been used. A property of the utility is that it is unaffected by a linear change. The process is not influenced by the reference points C and c: the suitable decision does not change with a varying origin or scale of utility.

It is assumed that the optimal choice is the option for which the expected value of the utility function is largest. If this assumption is accepted, theory can be used to predict the choice that the decision-maker should make among the set of possible actions. Therefore, decision theory gives a disciplined way of considering problems of decision and inference that offers the possibility of a rational choice in the presence of risk. For example, suppose the unknown 'states of nature' are θ_1 and θ_2 with current probabilities $\Pr(\theta_1)$ and $\Pr(\theta_2)$, respectively, such that $\Pr(\theta_1) + \Pr(\theta_2) = 1$. Assume there are two and only two possible decisions d_1 and d_2: choose d_1 if we believe θ_1 to be true and d_2 if we believe θ_2 to be true.

Let $u(d_1, \theta_1)$ be the utility of taking decision d_1 when θ_1 is true and define the other utilities similarly. The theory of maximization of the expected utility states that we should take decision d_1 if $E(U|d_1) > E(U|d_2)$. This will occur if

$$\begin{aligned} &u(d_1, \theta_1) \Pr(\theta_1) + u(d_1, \theta_2) \Pr(\theta_2) \\ &\quad > u(d_2, \theta_1) \Pr(\theta_1) + u(d_2, \theta_2) \Pr(\theta_2) \end{aligned}$$

which can be rearranged to give

$$\frac{\Pr(\theta_1)}{\Pr(\theta_2)} > \frac{u(d_2, \theta_2) - u(d_1, \theta_2)}{u(d_1, \theta_1) - u(d_2, \theta_1)} \tag{1}$$

This inequality has an intuitive explanation. The numerator on the right-hand side is $u(d_2, \theta_2) - u(d_1, \theta_2)$, the additional utility involved in taking the correct decision when θ_2 turns out to be the correct state. It could also be considered as the potential *regret*, in that it is the potential loss in utility when we erroneously decide on θ_1 instead of θ_2. The denominator similarly acts as the potential regret when θ_1 is true. Hence equation (1) says we should only take decision d_1 if the odds in favour of θ_1 are sufficient to outweigh any extra potential regret associated with incorrectly assuming θ_2 [(12) at 86].

As mentioned by Lindley (13):

> "The value inserted for the utilities and probabilities are in no sense correct and any other values wrong. They represent the decision-maker's individual preferences and may be modified by him. The only inviolate feature of them is their coherence.
>
> [. . .] The utility values must cohere with related quantities in other decision problems. A decision problem in isolation can have any values for utilities and probabilities. It is the coherence with other problems that constrains their values." [(13) at 157].

The consequence C_{ij} will be valued differently by different people. For example, if C_{ij} is a pair of first-row seats at a concert, a person who loves music will greatly value C_{ij}, whereas someone who doesn't care for music will not be very interested in C_{ij} [(14) at 25]. Your utility function can vary from situation to situation. For example, it may depend upon your current amount of savings.

The utility function is intended to describe how much a unit of something is worth. This value can be changed and so the decision-maker can explore consequences. Through sensitivity analysis, the decision-maker explores how different things would have to be for him to change his mind about the decision. This indicates how robust the decision is and points out areas where uncertainty is particularly problematic. Quantitative analysis must be viewed as an exploration of possibilities, not of hard predictions. However, the process of quantification may help to clarify the thinking and it provides a way of assessing which part of the model has a particularly large impact on the outcome and on determining how robust the preferred course of action is to other possibilities (15). This point will be illustrated through an example later.

In conclusion, it has been shown that there exists a unique way in which to make sensible decisions. In order to use this way, it is necessary that:

1. Uncertainties of the outcomes of events are quantified by probabilities;
2. Consequences of possible actions are quantified using utilities.

The decision-maker then takes the decision that maximizes the expected utility. These points are summarised by De Finetti (8) in the following terms:

> "Probability is the idea underlying our decisions under uncertainty, and, inversely, the decision theory is the operational basis to test our evaluations of probability and to refine the logico-psychological sense that guide us [. . .]." [(8) at 248]

Decision theory provides a useful framework to explore alternatives. It forces the decision maker (i.e., the scientist) to recognize that deciding not to take action is just as much a decision as deciding which action to take. It forces the decision maker to recognize that he may err either by taking an unnecessary action or by failing to take a necessary action. It helps to formalize and categorize the thinking to make sure that all relevant possibilities have been considered.

Utility as a Non-Negative Number

Decision theory is often presented and explained through a very simple example. Suppose, for example, that on a given occasion you face the decision of whether or not to carry an umbrella, and consider, for sake of simplicity, only two possible states, future rain and future shine (3,16). Another traditional example is the following. You need to decide whether to go on a picnic or stay home; call these decisions d_1 and d_2, respectively. Your utility depends on the weather. Imagine two possibilities: it rains or it shines. The two uncertain events are called θ_1 and θ_2, respectively.

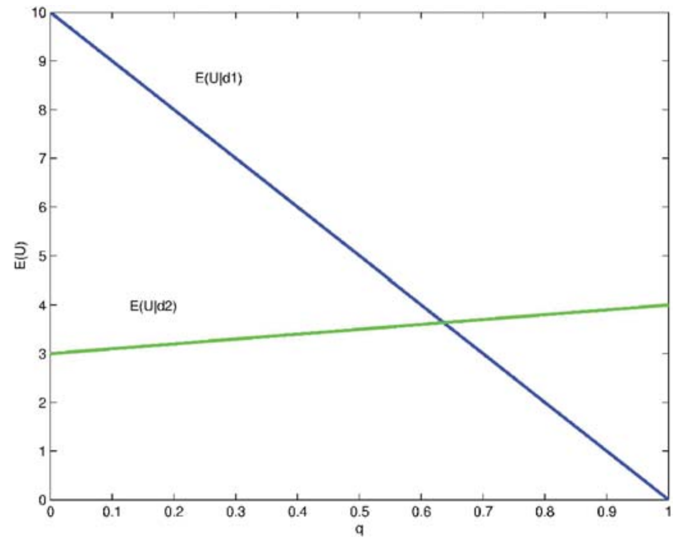

FIG. 1—*Expected utility as function of q where* $u(d_1,\theta_1)=0$, $u(d_1,\theta_2)=10$, $u(d_2,\theta_1)=4$, $u(d_2,\theta_2)=3$.

The utilities $u(d_i, \theta_j)$ are defined following a different scale from the previous section (note that there is a body of literature where new measures of utility have emerged, notably in the field of evidence-based medicine—see for example (17)—, but there is little thought on the assessment of utilities outside this field). In this example they are not probabilities. Here the best consequence is set equal to 10, the worst consequence is set equal to 0:

1. $u(d_1, \theta_1)$, go on the picnic and get wet: 0;
2. $u(d_1, \theta_2)$, go on the picnic and have fun: 10;
3. $u(d_2, \theta_1)$, stay home so you stay dry indoors: 4;
4. $u(d_2, \theta_2)$, stay home and it is nice: 3.

Let q be the probability of rain. The expected utilities for the two decisions are then

$$E(U|d_1) = q \cdot u(d_1, \theta_1) + (1-q) \cdot u(d_1, \theta_2) \qquad (2)$$

$$E(U|d_2) = q \cdot u(d_2, \theta_1) + (1-q) \cdot u(d_2, \theta_2) \qquad (3)$$

For $q = 0.1$, $E(U|d_1) = 9.0$; $E(U|d_2) = 3.1$. Expected utility as a function of q, the probability of rain, can be expressed graphically (see Figure 1). The optimal decision is to go on the picnic if $q < 0.634 \approx 0.64$. This threshold is defined by the intersection of the two utility functions.

Imagine you now hear a weather forecast. The forecast is for rain (call this event $F = r$) or for shine (call this event $F = s$). Knowing the sensitivity and specificity of the weather forecast you estimate the new expected utility as a function of the forecast, $F = r$ or $F = s$, where sensitivity is the probability it rains when it is forecast to rain and specificity is the probability there is sunshine when sunshine is forecast. It can be shown that collecting information is useful if it might change your decision. Here, the role played by inductive reasoning (*i.e.*, Bayesian) within the area of decision theory, illustrates the rational procedure to be followed in order to choose a decision in the best possible way. A *rational* actor will make those decisions that maximize (subjective) expected utility (or, equivalently, that minimize expected loss) [(16) at p. 4].

An example is presented in the Appendix using graphical models.

Case Study: The Kinship Determination

In addition to the parent-child determination in traditional parentage testing, other kinds of relationships of individuals also need to be tested in practice (18). Consider a situation involving the determination of kinship for possible inheritance consequences. Two individuals, say A and B, would like to know if they are full sibs or unrelated.

The two questions of interest (before performing the DNA profile test) in this scenario are:

1. Could the scientist obtain a value supporting the hypothesis H_p or H_d? Note that H_p and H_d are respectively defined as 'The pair of individuals A and B are full sibs' and 'The pair of individuals A and B are unrelated'.
2. How can the laboratory take a rational decision on the necessity to perform a DNA profile test?

Pre-assessment results (as presented by Cook et al. (2) in the glass scenario) allow the scientist to answer question 1. Decision theory deals with question 2.

Pre-assessment and Likelihood Ratios Distributions

Fundamental to this kinship scenario is the estimation of useful distributions for evidence under the two competing propositions. Allele frequencies (at different *loci*) from a selected population database are chosen. It is assumed that these frequencies can be used to create other databases as suggested by Triggs and Buckleton (19) through simulations techniques. One database of a large number of pairs of siblings and one of a large number of pairs of unrelated individuals are generated.

For a given couple of individuals in the first database (siblings), a likelihood ratio, V, is estimated

$$V = \frac{\Pr(G_A, G_B|H_p)}{\Pr(G_A, G_B|H_d)}$$

where G_A and G_B represent the genotypes of individuals A and B, respectively. The same procedure is performed for couples of individuals coming from the second database (unrelated individuals). A data set of 50,000 pairs of individuals (per database) is generated. Simulations have shown this number of pairs is sufficient to estimate the distributions of the likelihood ratios. A coancestry coefficient, F_{ST}, equal to 0.01 was used in simulations to take into account sub-populations effect in assessing random match probabilities (20).

So, two distributions of likelihood ratios are obtained. The first assesses full sibship for related individuals (brothers), the second assesses full sibship for unrelated individuals. Distributions are presented in Figure 2 (note that the natural logarithm of V is used to reduce the skewness).

Figure 2 shows that $\log(V)$ values are greater for full siblings than for unrelated individuals. There is an overlap between the two distributions. The overlap increases as the strengths of the relationship in the populations defined under H_p and H_d increase. Scenarios involving half-siblings versus unrelated and full siblings versus half-siblings are presented in Figs. 3 and 4.

Figures 2, 3 and 4 provide the answer to the question "could we obtain a value supporting the hypothesis H_p or H_d in this scenario?" Values of V show that an informative result can be obtained as is explained in the next Section.

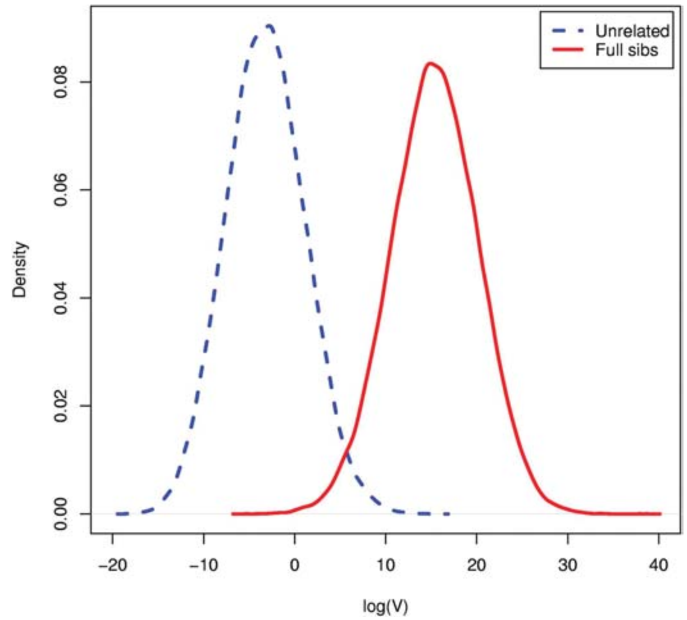

FIG. 2—*Distribution of the likelihood ratio, V, for full siblings versus unrelated.*

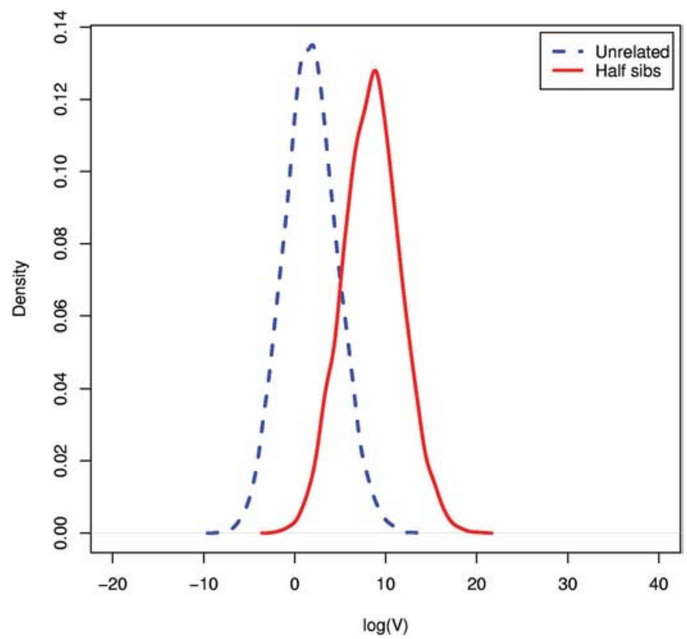

FIG. 3—*Distribution of the likelihood ratio, V, for half siblings versus unrelated.*

Posterior Probabilities

In the context of paternity and kinship, it is acceptable to make a digression from consideration solely of the likelihood ratio V and to consider the probability that individuals A and B are full sibs; *i.e.*, the probability that H_p is true. This probability is known as the *probability of sibship*.

Consider a piece of evidence, E_1, where $E_1 = \{G_A, G_B\}$ at a first *locus*. The odds in favour of H_p, given E_1 may be written, using the odds form of Bayes' Theorem, as

$$\frac{\Pr(H_p|E_1)}{\Pr(H_d|E_1)} = \frac{\Pr(E_1|H_p)}{\Pr(E_1|H_d)} \times \frac{\Pr(H_p)}{\Pr(H_d)}$$

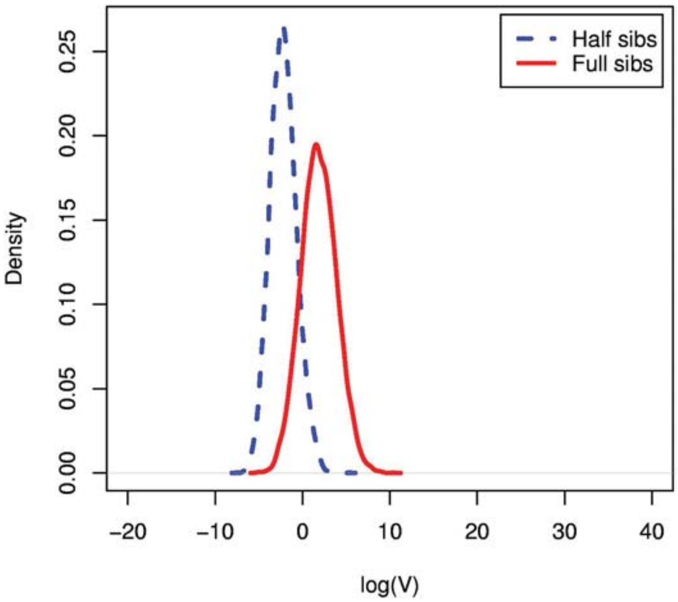

FIG. 4—*Distribution of the likelihood ratio, V, for full siblings versus half siblings.*

and $\Pr(H_d|E_1) = 1 - \Pr(H_p|E_1)$ so that, after rearrangement,

$$\Pr(H_p|E_1) = \left\{1 + \frac{\Pr(E_1|H_d)}{\Pr(E_1|H_p)} \times \frac{\Pr(H_d)}{\Pr(H_p)}\right\}^{-1} \tag{4}$$

Suppose (rather unrealistically) that the prior on the two propositions (each possibility) are equally likely. Then $\Pr(H_p) = \Pr(H_d) = 0.5$ and $\Pr(H_p|E_1) = 1/(1+V)$.

Now include $E_2, E_3, \ldots, E_n$ with E_1, thus describing results for *n loci*. The posterior odds in favour of H_p, given E_1, now replace the prior odds for the new evaluation. In general, for n independent DNA markers, giving evidence $E_1, E_2, \ldots, E_n$, with $\Pr(H_p) = \Pr(H_d)$,

$$\Pr(H_p|E_1, \ldots, E_n) = \left\{1 + \prod_{i=1}^{n} \frac{\Pr(E_i|H_d)}{\Pr(E_i|H_p)}\right\}^{-1}$$

$\prod_{i=1}^{n} \Pr(E_i|H_d)/\Pr(E_i|H_p)$is the product of the reciprocals of the n likelihood ratios $\Pr(E_i|H_p)/\Pr(E_i|H_d)$. This expression is also called the *plausibility of kinship*. Notice that it depends on the assumption $\Pr(H_p) = \Pr(H_d) = 0.5$. The assumption that $\Pr(H_p)$ equals $\Pr(H_d)$ may easily be dispensed with to give the following result

$$\Pr(H_p|E_1, \ldots, E_n) = \left\{1 + \frac{\Pr(H_d)}{\Pr(H_p)} \prod_{i=1}^{n} \frac{\Pr(E_i|H_d)}{\Pr(E_i|H_p)}\right\}^{-1} \tag{5}$$

The effect on the posterior probability of altering the prior probability can be determined from equation (5). The plausibility of kinship has also been transformed into a *likelihood of kinship* (i.e., likelihood of paternity) (21–22) to provide a verbal scale, given here on Table 2, columns 1 and 2.

Hummel's scale can be used to characterize states of nature (θ_j) in the decision approach as presented in the next Section, because legal decision in kinship cases is closely related to this scale by jurisprudence.

TABLE 2—*Hummel's scale and states of nature.*

Plausibility of Paternity	Likelihood of Kinship	States of Nature
Greater than 0.9979	Practically proved	θ_1
0.9910–0.9979	Extremely likely	θ_2
0.9500–0.9909	Very likely	θ_3
0.9000–0.9499	Likely	θ_4
0.8000–0.8999	Undecided	θ_5
Less than 0.8000	Not useful	θ_6

TABLE 3—*Decision table.*

	States of Nature					
Decisions	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
d_1	$u(C_{11})$	$u(C_{12})$	$u(C_{13})$	$u(C_{14})$	$u(C_{15})$	$u(C_{16})$
d_2			$u(C_{2\bullet})$			
$\Pr(\theta)$	$\Pr(\theta_1)$	$\Pr(\theta_2)$	$\Pr(\theta_3)$	$\Pr(\theta_4)$	$\Pr(\theta_5)$	$\Pr(\theta_6)$

TABLE 4—*Utility of consequences of decision 1 and 2.*

	States of Nature					
Decisions	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
d_1	1	0.5	0.1	0.1	0.1	0
d_2	0.7	0.7	0.7	0.7	0.7	0.7

Decisions and uncertain events

At first, a decision, its alternative and the uncertain events (states of nature) are defined. Decisions d_1 and d_2 are "Perform the DNA profile test" and "Not perform the DNA profile test," respectively. Uncertain events are specified using the states presented in the Hummel's scale. States of nature are specified from a particular point of view, the full sibblings perspective, where an individual is interested in proving the sibling relationship. On the contrary if an individual is interested in proving unrelatedness, then the order of Table 2 is inverted: θ_1 represents the state *Not useful*, and so on.

The decision to perform or not to perform the DNA test produces the following consequences:

- C_{11}: perform the DNA test and obtain the answer *Practically proved* (best consequence);
- C_{12}: perform the DNA test and obtain the answer *Extremely likely*;
- C_{13}: perform the DNA test and obtain the answer *Very likely*;
- C_{14}: perform the DNA test and obtain the answer *Likely*;
- C_{15}: perform the DNA test and obtain the answer *Undecided*;
- C_{16}: perform the DNA test and obtain the answer *Not useful* (worst consequence);
- $C_{2\bullet}$: do not perform the DNA test. Logically, if no test is performed, there can be no answer.

All these aspects of the decision problem are summarized in Table 3.

The expected utilities of d_1 and d_2 are, respectively:

$$E(U|d_1) = \sum_{i=1}^{6} u(C_{1i}) \Pr(\theta_i) \tag{6}$$

$$E(U|d_2) = u(C_{2\bullet}) \tag{7}$$

Utilities are obtained using the standard presented in the previous Section "Utility as a probability;" they are shown in Table 4. Utilities equal to 1 and 0 are assigned to states "Practically proven" and "Not useful," respectively, considered to be the best and worst consequences.

TABLE 5—*Probabilities of the states of nature for unrelated pairs.*

	Prior Probability of Sibship p								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
θ_1	0.0048	0.0079	0.0109	0.0138	0.017	0.0213	0.0268	0.034	0.0501
θ_2	0.0072	0.0099	0.0129	0.0157	0.0188	0.0224	0.0269	0.0345	0.0459
θ_3	0.0170	0.0248	0.0310	0.0385	0.0450	0.0518	0.0589	0.0699	0.0862
θ_4	0.0124	0.0183	0.0227	0.0253	0.0282	0.0317	0.0376	0.0408	0.0479
θ_5	0.0194	0.0258	0.0302	0.0338	0.0385	0.0427	0.0451	0.0510	0.0575
θ_6	0.9392	0.9133	0.8923	0.8729	0.8525	0.8301	0.8047	0.7698	0.7124

TABLE 6—*Probabilities of the states of nature for full siblings.*

	Prior Probability of Sibship p								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
θ_1	0.9261	0.9469	0.9575	0.9647	0.9702	0.9752	0.9804	0.9852	0.9905
θ_2	0.0338	0.0245	0.0204	0.0175	0.0155	0.0133	0.0105	0.0079	0.0052
θ_3	0.0219	0.0169	0.0130	0.0108	0.0088	0.0068	0.0056	0.0041	0.0028
θ_4	0.0060	0.0039	0.0033	0.0023	0.0020	0.0016	0.0011	0.0012	0.0006
θ_5	0.0040	0.0030	0.0019	0.0016	0.0010	0.0012	0.0010	0.0007	0.0004
θ_6	0.0082	0.0048	0.0039	0.0031	0.0025	0.0019	0.0014	0.0009	0.0005

TABLE 7—*Expected utility of d_1 and d_2, in the case of testing full siblings versus unrelated.*

Expected Utility	Prior Probability of Sibship								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$E(U\|d_1)$	0.9462	0.9615	0.9695	0.9749	0.9791	0.9828	0.9864	0.9898	0.9935
$E(U\|d_2)$	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7

Utilities u assigned to intermediate consequences C_{1j}, $j = 2, \ldots, 5$, are specified answering the following question:

> "does the decision-maker prefer the intermediate consequence or does he prefer the best consequence ('Practically proven') with probability set equals to u?"

For example, consider C_{13}. Does the decision-maker prefer to perform the test and learn that the kinship is very likely (plausibility between 0.9500 and 0.9909) or to accept that if he performs the test the result that kinship is practically proved (plausibility greater than 0.9979) will be obtained with probability 0.1 (the value of u assigned to θ_3 for d_1 in Table 4)?

The value u is the threshold of indifference.

The utility of the consequence under d_2 is fixed equals to 0.7: this means that it is assumed the decision-maker is indifferent between not performing the test and performing the test and obtaining the best consequence ("Practically proven") with probability 0.7. The utility of not performing the test is a direct consequence of the cost of performing it. This cost may be purely monetary. More generally it may combine monetary costs with other non financial burdens, such as inconvenience, intrusiveness, etc. Different decision-makers may state very different values for $u(C_{1j})$, $j = 1, \ldots, 6$ and $u(C_{2\bullet})$, depending on the kind of interest there is in determining the level of parentage, and on their adversity on risk.

Consider the propositions being tested are: full siblings and unrelated. Starting from the empirical cumulative distribution of the posterior probability of sibship, the probabilities of the states of nature $\theta_1, \ldots, \theta_6$ have been calculated for different values of the prior probability of sibship p, ranging from 0.1 to 0.9. Table 5 gives the probabilities of the states of nature calculated for pairs of unrelated people, while in Table 6 there are the probabilities computed for pairs of full siblings.

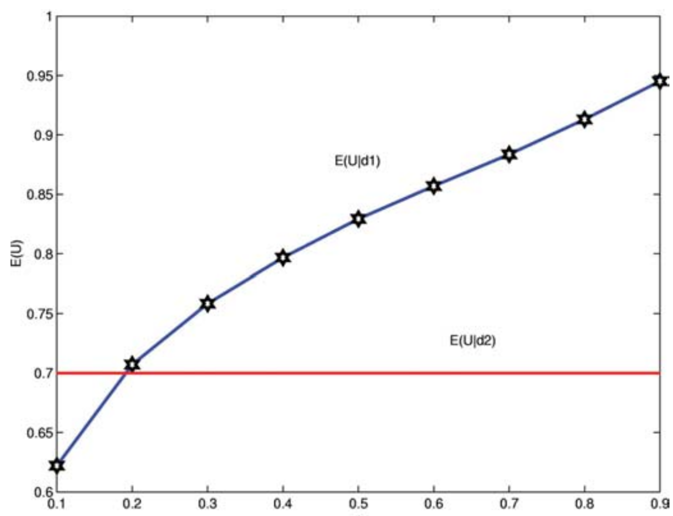

FIG. 5—*Expected utilities for d_1, d_2. The case of half-siblings versus unrelated pairs, for varying prior probabilty p of sibship.*

The expected utilities of d_1 and d_2 have been computed as in equations (6) and (7) for different values of p (see Table 7).

So, if a couple of individuals are really full siblings, the DNA test will generally confirm it and d_1 should always be used.

The situation becomes more complicated if the DNA test is requested to find out a different level of parentage, such as half-sibship versus unrelatedness or full-sibship versus half sibship. Figures 3 and 4 show that there is an increasing overlap among the distributions of the likelihood ratio, so there is a larger uncertainty.

Figure 5 shows the value of the prior probability of sibship p starting from which the expected utility of performing the test overlaps

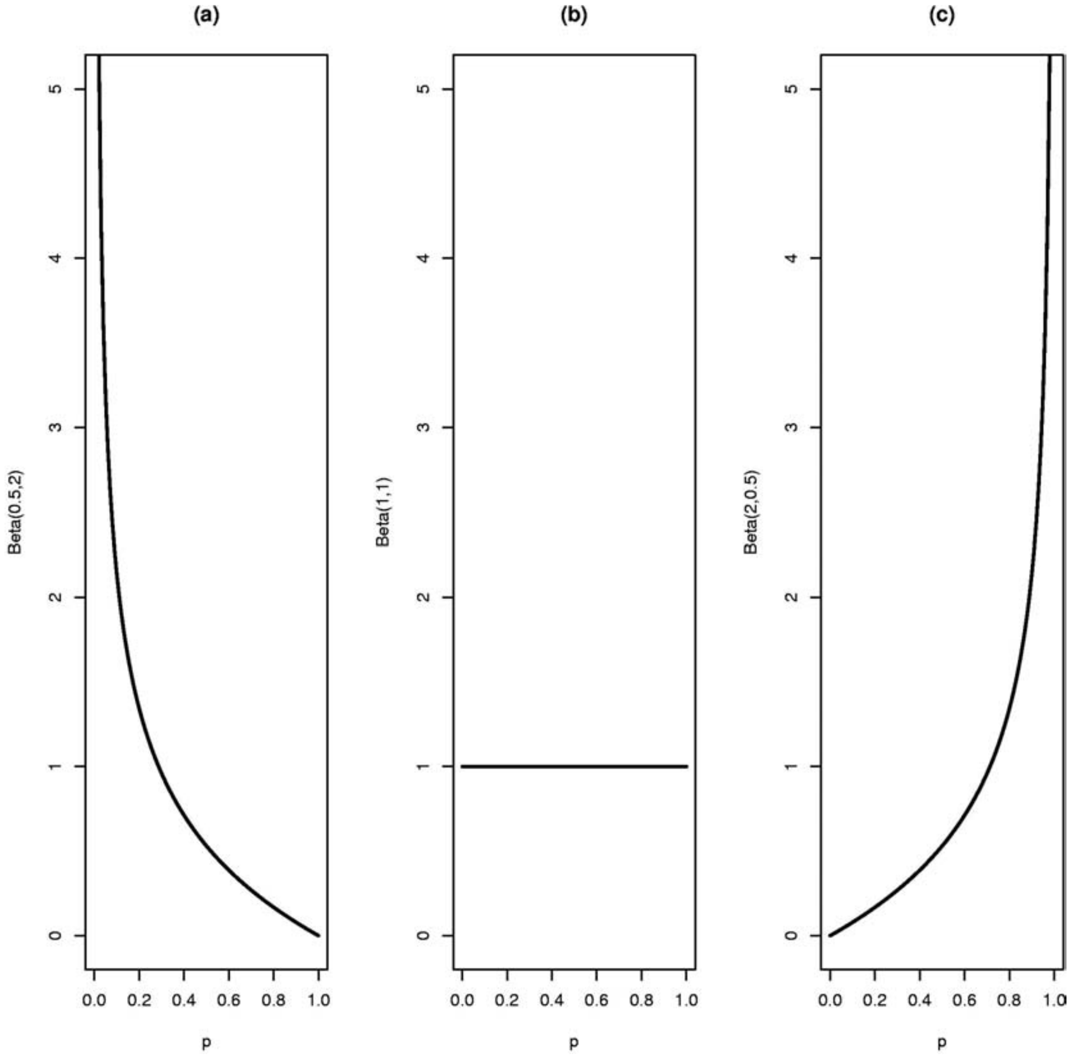

FIG. 6—*Probability densities on the prior probability of sibship p.*

the expected utility of not performing it, in the case half-siblings versus unrelated. So, if the decision-maker believes the Court will adopt a prior probability of sibship greater than 0.2, the test should be done. But what is the probability that the prior probability p will be greater than 0.2? There may be great uncertainty about the prior that will be adopted. Three different probability distributions are proposed here for illustrative purposes. They reflect different beliefs by the scientist on the value of the prior of probability of paternity p that will be adopted by the Court. The first one reflects a high belief on low values of the prior p, $p \sim Be(\alpha = 0.5, \beta = 2)$, the second one a situation of great uncertainty, $p \sim Be(\alpha = 1, \beta = 1)$, while the last one a high belief on high values of the prior p, $p \sim Be(\alpha = 2, \beta = 0.5)$, see Figure 6. Here $Be(\alpha, \beta)$ denotes the Beta distribution such that the probability density function $f(p)$ for p has the form

$$f(p|\alpha, \beta) = \frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha, \beta)}, \quad 0 < p < 1, \tag{8}$$

denoted $Be(\alpha, \beta)$, where

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)},$$

and Γ is the gamma function such that

$$\Gamma(x+1) = x! \text{ for integer } x > 0,$$
$$\Gamma(1/2) = \sqrt{\pi}. \tag{9}$$

See Aitken and Taroni (23) for further details.

In Table 8, the cumulative distribution functions, $F_1(\cdot)$, $F_2(\cdot)$, $F_3(\cdot)$, corresponding to the three situations, computed for different values of the prior p.

Suppose now that the distribution of the posterior probability of sibship suggests, given the preferences of the decision maker, that the analysis is convenient only if the Court will adopt a prior greater than $p = 0.2$. Table 8 gives, under three general situations, the probability that a value less than $p = 0.2$ will be adopted. In particular, if there is a strong belief that the Court will adopt a high prior probability of sibship (situation 3), it can be seen that $F_3(0.2) = \Pr(p \leq 0.2) = 0.016$. So, in this case it will be convenient to perform the test. Vice-versa, in situation 1, $F_1(0.2) = \Pr(p \leq 0.2) = 0.63$, there will be much more uncertainty.

It has already been pointed out that the final decision will depend on the preferences of the decision-maker, since utilities depend on them. Note that conflicts in decision-making arise because different

TABLE 8—*Cumulative probabilities of paternity priors for (a) Be(α = 0.5, β = 2), (b) Be(α = 1, β = 1), (c) Be(α = 2, β = 0.5).*

	Prior Probability of Sibship								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
(a) $F_1(p)$	0.46	0.63	0.74	0.82	0.88	0.92	0.96	0.98	0.99
(b) $F_2(p)$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
(c) $F_3(p)$	0.003	0.016	0.037	0.07	0.11	0.17	0.26	0.37	0.54

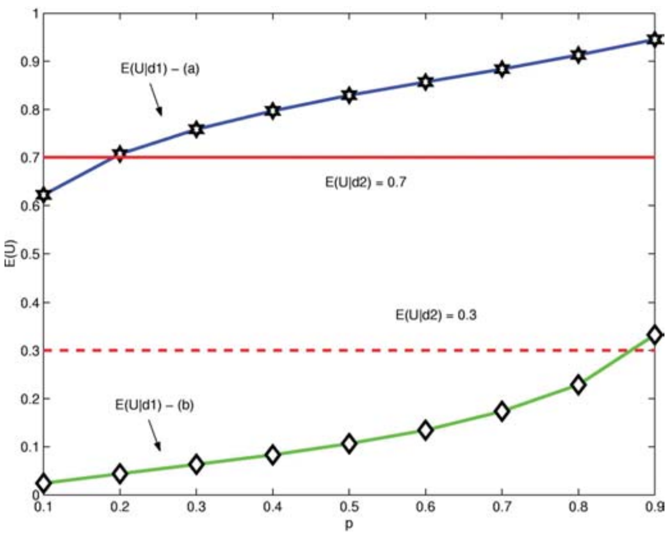

FIG. 7—*Expected utilities of d_1, d_2. Case half siblings versus unrelated (a), and full siblings versus half siblings, (b).*

decision makers place different values on different aspects of the outcome. This is not a problem because the framework uses subjective estimates, so different conclusions could be reached. The popular argument goes that if a probability represents a degree of belief, then it must be subjective in the sense of arbitrary, because personal beliefs could be different. This is wrong. Probability does indeed represent how much it is believed that something is true. This belief should be based on all the relevant information available; the information at someone's disposal may not be the same as that accessible to anyone else. This is not the same as arbitrary; it simply means that probabilities are always conditional, and this conditioning must be stated explicitly.

Sensitivity analysis can be performed, to analyze how sensitive the model is to changes in parameters or inputs where the output is the choice of action. The changes to be tested may be variations in the evidence provided or variations in the parameters, especially probability tables or utilities. A simple example is presented in Fig. 7 where utilities associated to decision d_2 are modified. Curve (a) represents the expected utility computed for different values of the paternity prior p when testing half-sibship versus unrelatedness, while curve (b) represents the expected utility when testing full-sibship versus half-sibship (the utilities of consequences of d_1 and d_2 are stated as in Table 4). So, if for instance $u(C_{2\bullet})$ is set equals to 0.7, d_1 would be convenient in case (a) only if the prior of sibship was at least equals to 0.2, while in case (b) d_1 would never be convenient. Otherwise, if $u(C_{2\bullet})$ is set equal to 0.3, decision 1 would always be convenient when the hypothesis being tested is the one contemplated by case (a), while in case (b) decision 1 would be convenient only for very high values of the prior p, such as p greater than 0.9.

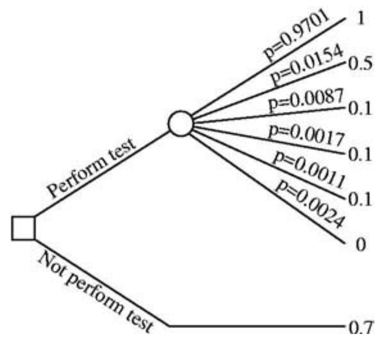

FIG. 8—*Decision tree in case of testing full siblings versus unrelated as in Section 'Decision and uncertain events'.*

Graphical Models

An inference problem can be broken down into smaller problems that can be solved separately and then combined to provide a solution to the larger problem (13). Scientists can solve the problems occurring at the level of scientific evidence and then use the rules of probability calculus to make them cohere with the entire evidential body. This is the concept behind graphical models. They allow the scientist to represent and reason about an uncertain domain of interest.

Graphical models provide a language of building blocks for constructing probability and decision models from modular components. They also provide a full picture of the problem under investigation. Through their use, it is hoped to make all of the implications of reasoning clear to lay people, without them having to understand any of the underlying mathematics or how to perform any calculation.

Probability trees are presented first. Then, influence diagrams are introduced. These are directed graphs representing the options, probabilities of the consequences, and the utilities of the consequences in a decision problem.

Probability Trees

A decision tree is a graphical model characterized by two kinds of nodes:

- the chance (or uncertainty) nodes representing random variables (circle in Fig. 8),
- the decision nodes representing decisions to be made (square in Fig. 8).

The branches emanating from a square correspond to the choices available to the decision maker, and the branches from a circle represent the possible outcomes of a chance event. The third decision element, the consequence, is specified at the ends of the branches. The utility of the outcome is the value of the outcome to the decision-maker.

Values associated with each possible outcome and probabilities associated with each branch of the tree are specified. Thus, it is possible to calculate the expected value associated with each possible decision. The expected value of a decision is the weighted average of all outcomes associated with the decision, where the weights are the probabilities associated with each step in the decision tree.

The problem described in Section "Decision and uncertain events" in the case of testing full siblings versus unrelated (in the specific case of $p = 0.5$) can be represented by a decision tree. The tree is shown in Fig. 8.

A decision tree represents all of the possible paths that the decision maker might follow through time, including all possible decision alternatives and outcomes of chance events. It is sometimes useful to think of the nodes as occurring in a time sequence. Beginning on the left side of the tree, the first thing to happen is typically a decision, followed by other decisions or chance events in chronological order. So, placing a chance event before a decision means that the decision is made conditional on the specific chance outcome having occurred (24).

Bayesian Networks and Decision Networks

Nodes and arcs are the main ingredients of a Bayesian network (BN). Nodes represent a set of random variables. Each node is characterized by states describing the values the corresponding variables can take. A set of directed arcs connects pairs of nodes representing the direct dependencies between variables. The strength of dependencies between variables is quantified by (conditional) probability distributions associated with each node.

BNs provide a valuable aid for representing relationships among characteristics in situations of uncertainty. They assist the user not only in describing a complex problem and communicating information about its structure but also in calculating the effect of knowing the truth of one proposition or one piece of evidence on the plausibility of others. BNs represent uncertainty and may be used for probabilistic inference.

Forensic science and judicial literature has already underlined that complex frameworks of circumstances, notably situations involving many variables, require the assistance of a logical approach (25–26). Methods of formal reasoning have been proposed to assist the forensic scientist to understand all of the dependencies which may exist among different aspects of the evidence (27–33). The use of graphical models to represent legal issues is not new (34). Probabilistic networks have been reintroduced recently with the analysis of complex and famous cases such as the Sacco and Vanzetti case (35) and the O. J. Simpson trial (36).

An extension of BNs allows the scientist to obtain an aid to support decision making. Adding an explicit representation of the decisions under consideration and the value (*utility*) of the resulting outcomes (the states that may result from a decision, also called *action*) gives *(Bayesian) decision networks* (BDNs). BDNs combine probabilistic reasoning with utilities to make decisions that maximize the expected utility.

A BDN consists of three types of nodes:

- the *chance* nodes representing random variables (as in BNs),
- the *decision* nodes which have a rectangular shape and represent the decision being made at a particular time and the utility nodes. The value of a decision node are the actions that the decision maker must choose between,
- the *utility* nodes which have a diamond shape and represent the decision maker's utility function. They are characterized by utility tables specified for every variable describing the outcome state that directly affect the utility.

Figure 9 represents the BDN for the genetical problem presented in Section 'Decision and uncertain events' in the case of testing full siblings versus unrelated (in the specific case of $p = 0.1$).

Figure 9 illustrates that the decision (d_1 or d_2) and the chance node influence the utility node as expressed by the arcs from those nodes to the utility node. The utility node describes the value of the consequence C_{ij} and the number associated with it is denoted $u(d_i, \theta_j)$. Values are specified in Section "Decision and uncertain events."

The probability table associated to the chance node presents probabilities for $\theta_1, \ldots, \theta_6$ obtaining through simulation techniques as presented in Section "Decision and uncertain events."

Note that there is no parent node to the decision node (no arc pointing to this node). In fact, no state of nature is known at the time the decision is made so no link is requested. A different situation is presented in the Appendix.

The network allows the decision maker to check expected utilities for the two decisions to be able to choose the more suitable decision. Graphical models represent, in an economic, simple and intuitive way, the probabilistic relations existing among the variables in a decision situation.

Decision trees and BDNs are complementary in the sense that the latter are particularly valuable for the structuring phase of problem solving and for representing large problem. Decision trees display the details of a problem; they display considerably more information than do decision networks where information is compacted into nodes. Decision networks are generally preferred because their understanding in a decision analysis is superior, regardless of mathematical training. Graphical models allow the scientist (the decision maker) to approach a problem, to structure it, to solve it without taking account of mathematical background.

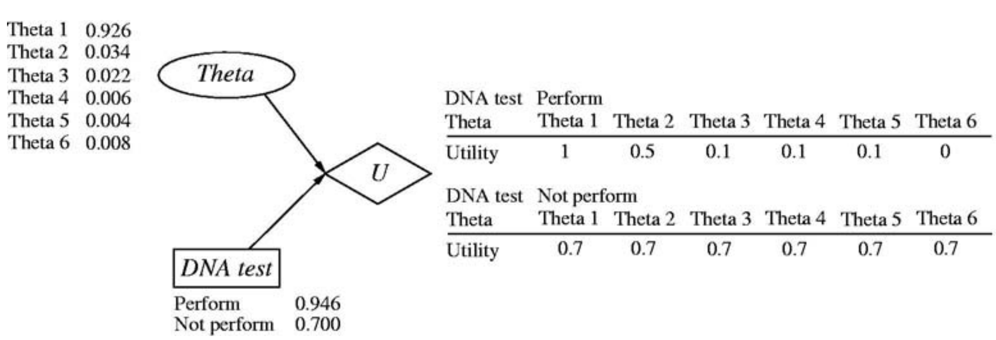

FIG. 9—*Kinship problem represented by a decision network.*

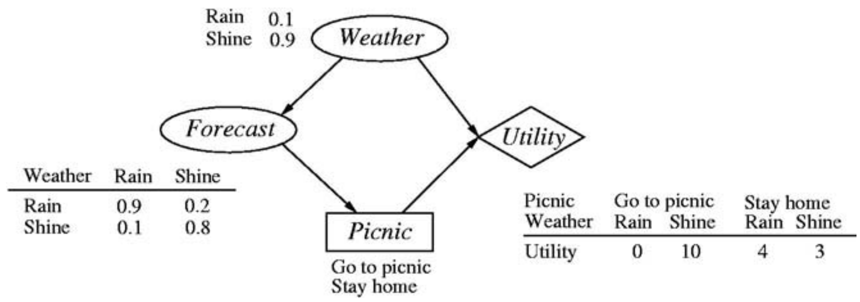

FIG. 10—*Picnic problem represented by a decision network.*

Conclusion

All forensic scientists have to cope with uncertainty. They have to take into account: (a) theoretical uncertainty, because no complete theory is known about the problem domain (e.g. police investigation, forensic examination), (b) cost-effectiveness, because the space of relevant factors can be very large and would require too many resources to investigate fully, (c) practical uncertainty, because there is uncertainty about a particular individual in the domain of interest, and (d) decision-making under uncertainty, because it is necessary to make rational decisions even when there is not enough information to prove that a proposed course of action will work. This point has been emphasized through the use of decision theory.

Decision theory is a body of knowledge and related analytical techniques designed to help a decision maker choose amongst a set of alternatives in light of their possible consequences. It is a mathematical theory of how to make decisions in the face of risk and uncertainty.

Graphical models have been introduced. They allow the forensic scientist to: (a) think about the problem involving uncertain information, (b) learn how to apply these methods to draw inferences about the world of interest, and (c) learn how to act rationally under risk.

Decision theory and graphical models will offer forensic scientists new tools to approach and interpret complex patterns of evidence. They will guide them in their decision process, focus their resource into the appropriate data gathering process and provide them with a intuitive tool to expose their thinking. Despite the cognitive difficulties (the representation and communication of values and utilities), decision-oriented quantification is an almost indispensable component of good forensic decision-making. These models will be essential in the day-to-day work of forensic experts and will ease communication between scientists and with other parties in the criminal justice system.

Acknowledgments

It is a pleasure to thanks Chris Triggs and John Buckleton for helping in the development of the breeding algorithm (code) used to create databases.

Appendix

Recall the "Picnic" scenario. You need to decide whether to go on a picnic or stay home; call these decisions d_1 and d_2, respectively. Your utility depends on the weather. Imagine two possibilities: it rains or it shines. The two uncertain events are called θ_1 and θ_2, respectively. The utilities $u(d_i, \theta_j)$ have been defined in Section "Utility as a non-negative number."

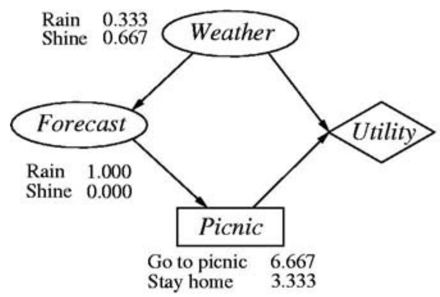

FIG. 11—*Picnic problem represented by a decision network with node 'Forecast' instantiated.*

The utility node is influenced by the chance node 'Weather' and the decision node. This is done by two arcs from those nodes to the utility node. The probability table linked to the new node "Forecast" expresses the forecast performance depending on node "Weather." Sensitivity and specificity of the forecast are presented in the first and fourth case of the table.

What is the optimal decision if the probability, p, of rain is 0.1? The expected utility for the two decisions are 9.0 and 3.1, respectively. This is automatically shown in the decision network (see Fig. 10).

The decision node has the chance node "Forecast" as parent. In fact, the value of the parent is known at the time the decision is made; hence the arc represents the sequence from parent node to child node. Now, imagine you learn new information about the weather. The forecast is for rain. This new information does not change your rational decision. In fact, the expected utility on d_1 is reduced from 9 to 6.66 as presented in Fig. 11 when node "Forecast" is instantiated. In the same time, the probability of rain, p, is updated, $\Pr(Weather = rain | F = r)$: from a prior of 0.10 to a posterior of 0.33.

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